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(*a*) Is everything in this world of ours completely determined by antecedent conditions, and is there no such thing in nature as an essentially unpredictable occurrence?

(*b*) Is every choice, every decision, of our lives determined by a definitely unbalanced condition among all of the influences which weigh for and against the choice or decision, and is there no such thing as essential freedom of the will and therefore no such thing as moral responsibility among men?

Of the two questions *a* and *b*, it seems that *a* is the more important because it is simpler, and because, as Huxley says "Nobody doubts that, at any rate within certain limits, you can do as you choose," and no one but a narrow fatalist or blind dialectician can doubt the reality of our moral responsibilities. The strongest evidence of the essential freedom of the will is our nâive sense of freedom, and the thing which nowadays tends more than anything else to discredit this nâive sense is the almost universal sophistication of latter-day civilized men in matters relating to the systematic sciences. If we except a small amount of traditional fatalism among sectarians in religion as of no consequence it would seem that the only strong presumption against the essential freedom of the will comes from an almost universal bias regarding question *a*.

This essay is devoted chiefly to question *a* in its application to lifeless nature. As the main purpose of the essay is to set forth a point of view which will force us to admit that something essentially unpredictable may occur tomorrow it is highly important at the start to recognize and to some extent to understand the universal philosophical bias which gives to everyone an instant answer to question *a;* "Yes, everything in lifeless nature is completely determined by an-

[1] The title of this paper as read before the Academy was Determinism in Biology and Indeterminism in Physics. This title was intended to convey the idea that the only determinisms in the world of actual happenings are the determinisms of genetics and physiology. The "equilibrium" world of the classicist in physics is almost as nothing in contrast with the biological world, and the perfect determinisms of this "equilibrium" world are of a lower order than the real determinisms in the biological world.

The paper has been revised since its delivery, and the author is greatly indebted to the outgoing chairman of the Publication Committee for the many alterations and additions made since January 1924.

tecedent conditions, and there is no such thing in lifeless nature as an essentially unpredictable occurrence." I have yet to find a single person who has returned other than this answer to question *a;* even those who persistently withhold an affirmative answer to question *b* have become so completely sophisticated in matters relating to the formal aspects of the physical sciences that they instantly accept an affirmative answer to question *a*.

What is here referred to as a philosophical bias may seem to be much more than that, for many scientists have come to believe in the existence of an objective principle or condition of universal validity, a principle which Helmholtz called "die Gesetzmäsigkeit der Natur." Of course Gesetzmäsigkeit is a fact, otherwise there could be no such thing as science, but it may not be of universal validity; a complete philosophy of nature may have to contemplate something beyond Gesetzmäsigkeit, something essentially unpredictable, lawless. At any rate it is the purpose of this essay to make this general point of view thinkable.

A Philosophical Bias.

What then is this philosophical bias and what is its source? To frame a tentative answer to this question let us note that the world in which we live is extremely complex and let us consider what characteristic of human psychology and what mode of human thinking would seem to be demanded for purely practical reasons in an extremely complex world.

Psychological. It is difficult to imagine an effective human psychology other than that with which we are familiar in which every sense perception is completed, as it were, by all but overwhelming subjective contributions so as to drown out extraneous and conflicting material, and lead, even in the face of infinite complexity, to a clearly defined behavior reaction. Otherwise there could be no behavior, or at best, like Brer Rabbit, we could only rush around! The subjective strengthening of one particular aspect or phase of a sense-complex until this aspect or phase stands out alone and becomes completely dominant, this seems to be the necessary characteristic of human psychology.

This characteristic of human psychology seems to be closely connected with what has been called the "all or none condition" in physiological psychology. A particular mode or phase of nerve activity may be likened to the explosion of a complex layout of powder or dynamite. The layout explodes completely or not at all. There is

no such a thing as a partial explosion and there seems to be no such thing as degree or intensity of nerve action. This is the "all or none condition" expressed in simple concrete terms. In a strong vigorous man the "complex layout of powder or dynamite" is quickly and completely repaired after an "explosion" and this quick and complete repair makes for sustained nerve activity, but gradations of nerve activity as respondent to gradations of external stimuli do not exist. Sensations which we describe as more or less *intensive* must be merely more or less sustained in time or more or less widespread in our nervous system, or, in other words, merely more or less *extensive;* and the above described strengthening and control of a sense-complex by subjective contributions must be dependent on *inhibitory action* on the one hand and on *inciting action* on the other hand, inhibitory action being the avoidance of certain aspects of a nerve discharge by back firing as in the control of a prairie or forest fire, and inciting action being the mere leaving open of certain bridgeheads or perhaps the laying of certain bridgeheads with powder trains. Of these two phases of control inhibitory action is certainly the more thinkable and perhaps real inciting action does not exist.

This mechanistic conception of psychological processes is of course very crude and probably all wrong, but it is a conception and therefore usable in thinking of this complicated business; and a very old idea receives a strong emphasis from this mechanistic conception, namely, that however completely the behavior of a human being may be correlated with the world in which he lives, his psychology must consist of elements so purely symbolic of world conditions themselves as to be hopelessly inadequate as a basis for any really adequate philosophy of nature. This old idea gets badly worn by neglect, and it fails to limit any man's philosophy; we ourselves possess a mysterious kind of curiosity which breeds a philosophic optimism that laughs at limitations! We do, however, wish to use this old limiting idea somewhat drastically, but we confess that we do not ourselves really submit to it.

It would seem that the subconscious contributions to behavioristic trains in psychology, however omnipotent, must be extremely rudimentary or animal-like and of little significance in our philosophy. Let us therefore, neglect these subconscious contributions, and we have left an almost complete rational control of behavior in response to sense impressions. This rational control is embodied in ideas, and postulates, and proverbs, and articles of belief. Recall if you can an idea or postulate or proverb or article of belief which does not bring to mind a definite thing or condition as an antecedent and an-

other definite thing as a consequence, or which does not tie two or more definite things together. We think only of things as tied together, we think only in terms of one-to-one correspondences. This is the philosophic bias to which we have referred, and it is putting the case mildly to call it a philosophic bias when it is the essential substance of our philosophic method, and indeed it has hitherto been nearly the whole of our method in science.

Mode of thinking. A good example of human thinking is furnished by the gardner who plants in the dark of the moon. There is a vast amount of human thinking of this gardener's kind, and it is essentially as reasonable as other modes of thinking for which we have less impatient contempt, for the gardener does plant and his success is such that he does not need to rectify his formula; therefore, let no one begrudge him his planting cue, *for cue he must have.* But a parlor discussion of the dark-of-the-moon idea is another thing! Once the writer was dreadfully oppressed by the apparent lack of qualifying humor in a long-drawn out and highly specific discussion of this idea, as it were a fact, but where no experimental use of the idea ever had or ever would be made; when, bursting out with Faust in the Witches' Kitchen:

"Mich dünkt ich hor ein ganzes Chor
Von hundert-tausend Narren sprechen"

the spell was broken because one other of the assembled company understood and appreciated Goethe's German! But the essence of human thinking is doubtful or wholly unjustifiable postulate and quick conclusion, and to conclude is to come to an end!—unless the conclusion is put to the test of experiment. Without the test of experiment our modern research institute would be a witches' kitchen ten thousand times confounded. This digression is intended to suggest the weakness of human reasoning when not continually rectified by critical appeal to experience, a weakness that is by no means so widely appreciated as it should be.

Natural Philosophy. Only two phases of natural philosophy have ordinarily been stressed in recent times, namely, (*a*) the philosophy of continua as exemplified by nearly the whole of modern mathematics, and (*b*) the philosophy of natural law in the mode of Newton and Maxwell as exemplified by nearly the whole of modern physics. These two aspects of natural philosophy may be called the classical natural philosophy for convenience.

a. Experimental. A very remarkable fact concerning the physical

sciences is that every laboratory study of a physical condition or thing involves the bringing of that condition or thing into an unchanging state by laboratory control. Classical physics is the study of permanent or quasi-permanent states.

A simple example of a piece of physical research is the work of Boyle and Gay Lussac, and I will set forth this example as if the whole of the experimental study of gases had been completed, as we now know it, by Boyle and Gay Lussac. They took a batch of gas and bottled it up and protected it as completely as possible from outside disturbing influences, "killing it dead" as it were, then subjecting it to very slow changes of volume and temperature, they measured volumes, temperatures and pressures with the utmost care. Thus they found definite one-to-one correspondences between the measured values of volume, temperature and pressure.

The equation $pv = RT$ expresses approximately[2] the laws of Boyle and Gay Lussac which are one-to-one correspondences between p, v and T, where p is the pressure, v is the volume, and T is the absolute temperature of a batch of gas, and R is a constant. This equation is applicable to the gas only where the gas is in thermal equilibrium. When the gas is not in thermal equilibrium it has no definite pressure or temperature, and its volume is, strictly speaking, a measurable property of the containing vessel which is to be thought of as in thermal equilibrium. If a slow reversible change of state of the gas is represented graphically in a pv, or pT, or Tv diagram, we get a continuous curve, but if at any stage the process becomes irreversible the gas becomes turbulent and there is a gap in the process curve. The important thing is to recognize that there are definite one-to-one correspondences of the Boyle and Gay Lussac kind which can be established with a degree of precision which is dependent only on the completeness of the control, on the precision[3] of the measurements, and on the use of a sufficiently large amount of gas.

[2] It is a common mistake to think of the classical laws of physics as *inexact* when they are merely *not simple* from the arithmetic or algebraic point of view. The simple algebraic expression of the gas laws, namely, $pv = RT$, is known to be only approximate, but there are very definite one-to-one relationships between p, v, and T when a considerable body of gas is kept in a quiet condition.

[3] Every physical measurement depends on a congruence operation, the fitting together of the thing to be measured and of the measuring structure; and errors of measurement (erratic errors of measurement) are of two distinct kinds, namely, *intrinsic* errors which are due to the inherent variability of the thing which is being measured, and *extrinsic* errors which are due to variability of the measuring structure. The so-called probable error of a measured result should therefore be separated into two parts, namely the *true probable*

All of the classical laws of physics are one-to-one correspondences between the various measured aspects of a system, they relate to substances or systems in permanent states, they are intrinsically approximate only when the permanence is incomplete or when the substance or system under test is so small that atomic or molecular irregularities become perceptible, and the essential narrowness of the classical method in physics has never been more strikingly pointed out than by Goethe, who, being a poet, could not be expected to recognize the necessity or appreciate the tremendous importance of a narrow prosaic point of view.

> "Wer will was Lebendiges erkennen und beschreiben
> Sucht erst den Geist heraus zu treiben."

Instead of bottling a batch of gas and wrapping the bottle in cotton wool, let it be turned loose and allowed to run wild! We can still follow it to some extent idealistically in terms of fleeting quasi-permanencies and roughly approximate one-to-one correspondences, and this is what your classicist in physics takes to be the whole story, but the point I shall wish to make is that a batch of gas turned loose and allowed to function in the forward movement of nature does perhaps run wild!

Quantitative chemistry, again, is not concerned with chemical actions themselves[4] but with their results. Consider, for example, a steam boiler plant, and fancy an engineer looking into the furnace and attempting to observe and record the height and breadth and momentary temperature of each flicker of flame! It is not done. The important thing is the amount of steam that is produced by each pound of coal, and this depends only on (1) the condition of the feed water from which the steam is made, (2) the quality of the coal before it is put into the furnace, (3) the initial condition of the air which enters the furnace, (4) the condition of the flue gases as they enter the chimney, and (5) the pressure and temperature of the steam which is

error which is due to variability of the measuring system and to the inaccuracies of congruence; and the *probable departure*, this departure being the probable departure of the measured system from the carefully considered estimate at any instant.

[4] In some cases chemical action shows itself, approximately as a permanently varying state, and the various aspects of a chemical system in a permanently varying state can be measured and correlated. Thus the slow saponification of an oil is approximately a permanently varying state and the study of slow saponification gives rise to the notion of reaction velocity. Permanently varying states of chemical systems constitute what the writer has called *steady sweeps*. See pages 128–148 of Franklin and MacNutt's *Heat*, Lancaster, Pa., 1923.

produced. That is to say, it is necessary to consider only the states of things *before* and *after* the combustion takes place, and the only measurements that need be made, the only measurements that can be made, are measurements of substances in thermal equilibrium. The laboratory method for the study of thermal and chemical changes is what may be, in mild derision, called the "before and after" method!

b. Methods. There are four distinct methods in physics, namely, the method of mechanics, the method of thermodynamics, the method of atomics, and the method of statistics; and all four methods come into play in the broad study of any particular condition or thing.

The method of mechanics is exemplified by what is narrowly called mechanics; under the name of generalized mechanics it includes the whole of the classical treatment of electricity and magnetism, of light and of sound; and it stops short where thermal and chemical changes come into consideration.

The method of thermodynamics is essentially an extension of the method of mechanics so as to include the study of thermal and chemical changes and this extension is based on two generalizations, namely, on the first and second laws of thermodynamics.

The methods of mechanics and thermodynamics are not to be apprehended by definition, but by actual use. *Both methods establish correlations directly in terms of measured data without any consideration whatever of the essential nature of physical conditions and things;* both methods are highly abstract and they neglect much that is by no means always negligible; both methods establish one-to-one correspondences between the various measured aspects of systems in permanent or quasi-permanent states; and what I have called the "before and after" scheme gives to the method of thermodynamics a truly remarkable range in the correlation of the consequences of irreversible processes or forward movements.

The atomic method is the building of elaborate conceptual pictures of the essential nature of physical conditions and things on a postulate foundation. The molecule, the atom, the ion, the proton, and the electron *as used in theory* are mere postulates in view of the fact that as physical things our knowledge of them is so incomplete.

Everyone is familiar with the use of the atomic method in elementary chemistry, and with the somewhat vague but nevertheless important use of the atomic theory in Tyndall's "Heat, a Mode of Motion." The so-called kinetic theory of gases is also quite well known, but the use of the atomic method in crystallography by

Sohncke and others before the days of Laue and the Braggs, and the elaborate attempt which was made a hundred years ago to use the atomic method in the theory of elasticity are not generally known.

The atomic method has been extensively used in the elucidation of previously known one-to-one correspondences. The kinetic theory of gases, for example, gives a remarkable rational insight into the laws of Boyle and of Gay Lussac. The atomic method has also been of use in the discovery of classical one-to-one correspondences, and the most remarkable example of this sort of use is, perhaps, Maxwell's discovery of the invariance of gas viscosity with respect to pressure when the temperature is constant. Furthermore an important use of the atomic method is in the rationalization of simple types[5] of forward movements such as reaction velocities, radio-activity, thermo-ionic emission, electrical conduction, etc.

The statistical method[6] has been used but little in physics, except in meteorology which I will consider later. The best known statistical studies in the laboratory are Perrin's study of the Brownian motion, Rutherford's study of the scattering of alpha and beta particles, and Millikan's study of the movement of charged droplets of oil through ionized air. All of these studies are intimately connected with the atomic theory in that they deal with atoms or something like atoms as the actual objects of study. Perhaps the greatest achievements of the atomic method in the future will be statistical—only then it will not be the atomic method as above defined for atoms themselves will be the physically real objects of study.

Helmholtz has used the word "systematik" to designate the methods and results of classical physics, which includes the methods of mechanics and thermodynamics and also the method of atomics when used as it has been hitherto in the discovery and elucidation of one-to-one correspondences. However, we often extend the classical systematic physics to systems which are very far removed from permanent states, but the ideas which are used, the measurements (approximate measurements) which are made, and the approximate one-to-one correspondences which are established are all borrowed from and interpreted in terms of ideal permanencies.

c. Idealization. It is necessary in what follows to refer repeatedly

[5] These are what the writer has called steady sweeps or permanently varying states which are amenable to measurement.

[6] What is here referred to is the use of the theory of probabilities in the arrangement and interpretation of observational material. The kinetic theory of gases and statistical mechanics (Gibbs) are postulate structures, they are not what is here called statistical physics.

to actual happenings in the physical world. Such happenings are so-called irreversible or sweeping processes, they always involve an increase of entropy,[7] and it is helpful to refer to them as *forward movements*.

Let us look very briefly into the most familiar branch of physics, namely, Mechanics. In the study of the performance of any kind of a mechanism a great variety of change is involved, and yet the science of mechanics is no exception to the general statement that physical science is the study of permanent or quasi-permanent states. To justify broadly this statement in its application to mechanics it would be necessary to show that every phase of laboratory control in any detailed mechanical study is for the sake of permanency; it would be necessary to explain at length the peculiarly idealistic[8] character of mechanical studies, in general, and it would be found that the superficial and idealistic character of the science of mechanics is bound up with and grows out of the fact that the irreversible aspects of physical happenings are completely ignored[9] in all mechanical studies. To undertake to do all this would lead us away from the main theme of this paper and it must suffice therefore to refer to the science of hydraulics as a partial illustration of the method of mechanics. A brief discussion of hydraulics is indeed worth while because the idealistic character of this branch of mechanics is easily explained and because everyone can understand the utter inadequacy of the idealized science of hydraulics on the basis of what can be easily seen in any actual case of fluid motion.

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[7] No generalization in physics is more important to the biologist than the "law of entropy" or the second law of thermodynamics. The elementary discussion of this law which is given in Franklin and MacNutt's *Heat* is recommended to the biologist because this discussion dwells at first on the physical facts, because the postulate character of the generalization itself is then set forth, because the meaning of the generalization is then exemplified in simple concrete terms, and because the mathematical arguments which lead to the experimentally verifiable consequences of the generalization are made as simple as it is possible to make them. Of all the generalizations of physics the second law of thermodynamics is the most deeply rooted in intuition.

[8] One of the most far reaching discussions along this line is an appendix in Sir Joseph Larmor's "Aether and Matter" on "The Scope of Mechanical Explanation." This appendix of Sir Joseph's is difficult to read, but the reader will be helped in getting a clear idea of what is here referred to by reading the simple discussion of the mechanical nature of force which is given on pages 174–178 of Franklin and MacNutt's "Electricity and Magnetism," Lancaster, Pa., 1924. This discussion is an exposition of the simpler ideas which are set forth in the above mentioned appendix to Sir Joseph's "Aether and Matter."

[9] Friction is indeed taken into account in many mechanical studies, but in a manner so utterly incomplete that this statement may be allowed to stand.

Hydraulics, as the term is here used, is the study of fluids in motion, and the phenomena of fluid motion are excessively complicated. The apparently steady flow of a river in a smooth channel is seen on close inspection to be an endlessly intricate combination of boiling and eddying motion. The jet of spray from a fire nozzle; the burst of steam from the safety valve of a locomotive; the fitful motion of the wind as indicated by fluttering leaves and quivering blades of grass, and as actually visible in driven clouds of dust and smoke; and the visible sweep of the flames in a conflagration—these are actual examples of fluid motion and they are indescribably, infinitely complicated! Everyone concedes the idea of infinity which is based on abstract numbers, one, two, three, four, and so on *ad infinitum*, and the idea of infinity which comes from the contemplation of a straight line; but most men are so narrowly concerned with the humanly significant and more or less persistent phases of the material world that their perception does not penetrate into the substratum of erratic action which underlies every physical happening, and they balk at the suggestion that the phenomena of fluid motion, for example, are infinitely complicated. Surely the abstract idea of infinity is nothing in comparison with the awful intimation of infinity which comes from things that are seen and felt.

The science of hydraulics is based on ideas which relate to average aspects of fluid motion. We can determine the time required to draw a pail of water from a hydrant or the drop of pressure along a pipe line from a pump to a fire nozzle, or the force exerted by a jet on the buckets of a water wheel; but such things are never perfectly steady even when the ruling conditions are completely controlled, they are always subject to perceptible variations of an erratic character, and to think of any of these effects as definitely quantitative is, of course, to think of their average character under the given conditions.

The most important conceptual element in the science of hydraulics is the idea of *simple flow*, and if we limit ourselves to what is called permanent or steady flow this idea of concept is that the velocity of the fluid at each point remains constant in value and unchanging in direction so that the motion of a fluid can be visualized in terms of what are called stream lines. Simple flow is however, an idea, not a fact; and stream lines never exist in actual fluid motion. Nevertheless, the whole of the science of hydraulics is based on the idea of simple flow and one or two additional sharply defined ideas just as the science of geometry is based on sharply defined ideas of which the most familiar is the idea of a straight line.

Now it is not the purpose of this paper to discuss the very great importance of precise ideas in the physical sciences nor to justify the useful and even necessary habit among physicists of accepting and talking about such ideas as facts, a habit which is similar to that of the ordinary man who thinks of a straight line as a physical fact rather than as a mere idea. Any other point of view involves too many qualifications in speech and thought, and qualifications always weaken speech and thought. It is a great help, says Helmholtz, to form the most concrete possible pictures of abstractions even when such pictures involve assumptions that are in all strictness unnecessary. It is not my purpose, I say, to point out the importance of precise ideas, but it is quite necessary for me to call your attention to the fact that the many precise ideas which underlie the physical sciences operate not only as a stimulus to, but also as a constraint on, the imagination.[10]

What would you say of a fire chief who might become absorbingly interested in the infinite complex of minute detail of the stream of spray from a fire nozzle? Of a farmer who would employ an innumerable squad of disembodied spirits who could be stationed in his fields without interfering with the drops of rain and set observing and recording the size and shape and placing of each individual drop of rain? And yet it is on minute details of this kind that I ask you to fix your attention and exercise your imagination, because it is the aim of this paper to show that any infinitesimal detail or element in the infinitive aggregate of such that exists in any forward movement of nature may conceivably dominate the movement as a whole and determine finite consequences.

[10] Reference may be made to an essay of C. S. Peirce, *The Monist*, April, 1892, reprinted in his Chance, Love and Logic (edited by W. R. Cohen), 1923. A quotation: "I propose here to examine the common belief that every single fact in the universe is precisely determined by law. It must not be supposed that this is a doctrine accepted everywhere and at all times by all rational men. Its first advocate appears to have been Democritus, the atomist, 'who was led to it, as we are informed, by reflecting upon the 'impenetrability, translation, and impact of matter.' That is to say, having restricted his attention to a field where no influence other than mechanical constraint could possibly come before his notice, he straightway jumped to the conclusion that throughout the universe that was the sole principle of action,—a style of reasoning so used in our day with men not unreflecting as to be more than excusable in the infancy of thought. But Epicurus, in revising the atomic doctrine and repairing its defences, found himself obliged to suppose that atoms swerve from their courses by spontaneous chance; and thereby he conferred upon the theory life and entelechy."

The Real State of Affairs.

What I have to say to you about indeterminism in physics is simple enough and it will take only a short time to tell; it has already been adumbrated in the discussion of idealization, but I must count on your continuing to take a realistic view of physical things, and by realistic I mean a freely imaginative sensuous view as opposed to a constrained aperceptive view which sees only what is in conformity with accepted ideas. For we must certainly break away from the idealization of classical natural philosophy if we are to give adequate attention to the discontinuous and to that which in particular is lawless and becomes lawful only in general.

The attempt is made in this paper to show that, conceivably, there are gaps in nature wherein there are no physical or chemical laws, no one-to-one correspondences expressible in terms of differential or integral equations, and that the finite consequences of such a gap or break are indeterminate.

Turbulence. The more an experimental system departs from an ideal permanent or permanently varying state, the more clearly we recognize an underlying erratic action which is indescribably complex. This erratic action exists everywhere and in everything that goes forward in nature, we call it *turbulence,* and I think it may be assumed to be infinitely complicated and never twice alike in detail even when the large-scale conditions are exactly the same. We have a macro-physics, a large scale physics, which is the physics of the classicist, and we have a new micro-physics, a small scale physics. The terms macro-physics and micro-physics are however misleading because turbulence often shows itself in large-scale phenomena such as storm movements in our atmosphere and storm movements in the sun, to say nothing of such frequent large-scale turbulences as automobile and train wrecks and collapsing structures! It is these large-scale turbulences which I wish especially to consider, not because they are essentially different from omnipresent small-scale turbulences, but because they are humanly significant and therefore real to everybody, and the point I wish to make is that *a large-scale turbulence can conceivably be attributed to infinitesimal causes.*

Turbulence is characteristic of those physical and chemical changes which are called irreversible or sweeping processes. Such processes involve increase of entropy and they are normally impetuous. The most familiar example of such a process is ordinarily fire, and, as everyone knows, a fire is not dependent on external driving cause but when once started it goes forward of itself and with a rush.

Impetuous processes, such as explosions and wrecks and the storm movements of our atmosphere, are intimately connected with conditions of instability. Indeed an impetuous process seems always to be the collapse of an unstable state. Let us therefore consider two ideal examples in which the condition of instability is supposed to be completely established at the beginning.

(*a*) Imagine a warm layer of air near the ground overlaid with cold air. Such a condition of the atmosphere is unstable and any disturbance however minute, may conceivably start a general collapse. Thus a grasshopper in Idaho might conceivably initiate a storm movement which would sweep across the continent and destroy New York City or a fly in Arizona might initiate a storm movement which would sweep out harmlessly into the Gulf of Mexico. These results are different surely, and the grasshopper and the fly may be of entirely unheard of varieties, more minute and insignificant than anything assignable. Infinitesimal differences in the earlier stages of an impetuous process may, therefore, lead to finite differences in the final trend of the process.

(*b*) Consider a smooth spherical ball traveling through still air. There certainly is no more reason to expect the ball to jump to the right than to the left. Therefore, we may conclude that it will not jump either way. Similarly, a sharp-pointed stick stands in a perfectly vertical position in a perfectly quiet room, and there is no more reason to expect the stick to fall one way than another: therefore the stick will not fall at all. Everyone appreciates the fallacy of this argument as applied to the stick, and the moving ball does, in fact, jump sidewise.

To understand the behavior of the ball, let us think of the ball as standing still and of the air as blowing past it in a steady stream. The air streams past the ball and slides over a body of still air behind the ball,[11] the surface which separates the moving air and still air is called a vortex sheet, and a vortex sheet is unstable. Any cause however minute, is sufficient to start an eddy or whirl, and once started such an eddy or whirl develops more and more. Such an eddy or a whirl means that the air streaming past one side of the ball is thrown inward or outward and the reaction on the ball pushes the ball sidewise. This effect can be shown by dropping a marble in a deep jar of water. Instead of moving straight downwards the marble follows an erratic zigzag path. This effect is familiar to everyone in the sidewise quivering of a stick or fishline in a stream of water; and

[11] The motion of the air is somewhat idealized in this statement.

the hissing of a jet of steam is due to the rapid fluttering of the boundary between steam jet and air because of the formation of innumerable eddies.

The above examples refer to the breaking of an unstable state by a minute disturbance. In the case of an ideal or complete instability the breaking disturbance can be smaller than anything assignable, that is to say, the breaking disturbance can be an infinitesimal disturbance in the strict mathematical meaning of this term, many of the finite characteristics of the resulting forward movement will be determined by the time and place of the infinitesimal disturbance, and we must admit that the idea of cause and effect is here inapplicable because a finite effect cannot be attributed to an infinitesimal cause.

A postulate. In that branch of mathematical physics which is called statistical mechanics and which includes the atomic theory, we speak of the complection of a system when we wish to refer to the positions and velocities of all the elements or particles of the system. Let us use this word in the statement of the POSTULATE OF INDETERMINISM. *The complection of the world tomorrow is not determinate*—that is to say, it does not grow out of the complection of the world today as a single-valued determinate thing. This is a postulate which, as it seems, must be accepted as a working hypothesis in the "extra-equilibrium" world, the world of actual happenings, where things never do stand still, but go forward by fits and starts impetuously and beyond all control.

Whenever the postulate of erratic action is set forth, and the probable departure of a natural phenomenon from the most carefully considered prediction is urged as in the nature of things inevitable, we meet objections from two classes of men, namely, the average man who thinks frankly in terms of human values (the gardener who is not concerned with individual drops of rain) and the classicist in science who idealizes nature in one-to-one correspondences. Surely the classicist says, "if we knew all the data we could make an unqualified prediction in any case." But, ignoring the hopelessly unscientific attitude of mind of one who can postulate infinite knowledge, let it be understood that to speak of data in physics is to speak of a very narrow and limited kind of thing, for data are conceivable only where measurements can be made or where we have, contrary to Bacon's exhortation, accepted a dream of fancy for a model of the world.

Meteorology has for many years been the only branch of physics dealing with actual uncontrolled happenings, and the statistical

methods which have heretofore been used in meteorology have been too much of the kind that is applicable to a homogeneous aggregate of erratic elements. Statistical meteorological studies have in most cases involved the leveling process of simple averaging, whereas the conditions would seem to require classification ten thousand times more exhaustive than any hitherto made. Rythmic variations of day and night, the succession of the seasons, bodies of land and water, mountains and valleys, and the influence of underlying regularities of atmospheric movement which show themselves in what we call prevailing winds, all these would seem to have much to do with the characteristics of incipient atmospheric instabilities and they must certainly operate as "grooves" as it were, in their influence on storm movements and determine a host of families and genera and species. Any adequate statistical study of a "grooved" system must use exhaustive classification studies. To be content with averages and probable departures derived from meteorological data uninfluenced by exhaustive classification studies is as hopeless as a study of plants and animals which ignores the existence of species.[12]

The conception of indeterminism, as dependent on the collapse of unstable states is decidedly incomplete in a realistic sense because every unstable state is an ideal. Static and dynamic instabilities of systems of rigid bodies are very highly idealized, and so also are static and dynamic instabilities of fluids; and all such instabilities of mechanical system are, perhaps, in their limits, incompatible with the atomic or molecular constitution of matter. However, the main purpose of this paper is to make indeterminism thinkable! Critical instabilities are highly idealized, as stated, but it does seem that an interpretation of nature which is based on the postulate that *critical instabilities do occur* is more realistic (on the whole less highly idealized) than the usual mechanistic interpretation which is based on the even higher degree of idealization which rules out the critical instability.

The postulate of indeterminism and its basis on instabilities is not new. It is set forth in a "practical" form in a remarkable paper by J. Boussinesq, entitled "Conciliation du Véritable Determinism Méchanique avec l'Existence de la Vie et de la Liberté Morale," Paris, 1878; reprinted in the third volume of his Theorie de la Chaleur, etc., Paris, 1922. That Boussinesq should incorporate this essay in his great treatise on heat and light shows that he still holds to his earlier

[12] For a further discussion of meteorology see my paper in the Monthly Weather Review, September 1918, in which paper is given a preliminary sketch of my point of view relative to indeterminism.

views, and it may indicate that he considers these views to be worthy of attention although they have been almost completely neglected hitherto. "Scientists are in agreement" says Boussinesq "that physical and chemical laws are reducible in the last analysis to differential equations" and indeterminism is connected with the behavior of differential equations at or near their singular points. The equations have several distinct solutions which differ only infinitesimally at or near a singular point; so that in passing through a singular point an infinitesimal influence is sufficient to determine the outcome of one or the other of the distinct solutions and thus lead to widely different consequences.

About the same time as Boussinesq, Maxwell was making a similar suggestion.[13] Says he: Every existence above a certain rank has its singular points, the higher the rank the more of them. At these points influences too small to be taken into account by a finite being may produce results of the greatest importance. All great results produced by human endeavor depend on taking advantage of these singular states when they occur. But singular points are by their very nature isolated and form no appreciable fraction of the continuous course of our existence. This view of Maxwell's makes indeterminism the exceptional, determinism the ordinary course of events, but the exceptional the more significant. An anology from biology might be mutation and from astronomy the cataclysmic change which leads to the birth of a double star.[14]

A Generalization. Although through the ideas of singular states and the collapse of unstable states, it may be possible to go some distance towards giving a rational theory and a convincing illustration of indeterminism in physics, we do not believe that these suggestions are adequate. So pervasive are the cases of observable indeterminism, so obvious does it appear that the whole mechanistic system and philosophy of systematic physics is an idealization directly away from intimate happenings for the purpose of simplifying microscopic explanation, that we prefer to regard the facts of nature as indicating indeterminism as not exceptional but general. A batch of gas turned loose and allowed to function in the forward movement of nature does perhaps run wild! We have, of course, chemical instabilities, and the recent developments of atomic theory, especially those which include

[13] See "The Life of Clark Maxwell" by L. Campbell and W. Garnett, 1882, pp. 434 ff.

[14] See for example, J. H. Jeans' Cosmogony and Stellar Dynamics, Chapter VI.

the quantum theory, seem to indicate the existence in nature of kinds of instability and of chance happenings previously undreamed.[15]

The point of view relative to indeterminism which we have tried to subsume in a postulate and to illustrate by pointing out what does seem to happen in the world of physics is cognate not so much to that of Boussinesq and of Maxwell as to that of C. S. Peirce. In an imaginary dialogue with an imagined opponent, after the style of Plato, Peirce says[16] "Very well, my obliging opponent, we have now reached an issue, you think all the arbitrary specifications of the universe were introduced in one dose, in the beginning, if there was a beginning, and that the variety and complication of nature has always been just as much as it is now. But I, for my part, think that the diversification, the specification, has been continually taking place." And again,[17] "An elememt of pure chance survives and will remain until the world becomes an absolutely perfect, rational, and symmetrical system in which mind is at last crystallized in the infinitely distant future."

Whether the last phrase is merely rhetorical for emphasis or whether it represents Peirce's notion of the inevitable line of evolution in nature's infinite variety is not clear, but with respect to evolution and the idealized laws of nature he does say this:[18] "Uniformities are precisely the sort of facts that need to be accounted for. Now the only possible way of accounting for these laws of nature and for uniformity in general is to suppose them results of evolution. This supposes them not to be absolute, not to be obeyed precisely. It makes an element of indeterminacy, spontaneity, or absolutely chance in nature." These are our *gaps;* they are inherent and not necessarily mere singular points in differential equations which cannot, as a matter of fact, be set up except on a basis of idealizations and uniformities.

[15] It is interesting to note that in the work of Einstein and his followers we find both determinism and indeterminism. On the one hand his generalized relativity with its developments to include electrical phenomena is the apotheosis of the Democritus-Newton-Laplace-Maxwell school of classical natural philosophy and has already found its interpreters among philosophers, notably A. N. Whitehead. On the other hand his quantum theory and theory of radiation with their experimental and theoretical consequences make a return to the Epicurean doctrine of the swerving of atoms (electrons and quanta are the prime movers now) from their courses by spontaneous chance; if such phenomena and notions become in due time thoroughly basic and permanent in physics, they, too, will find their interpreters among philosophers.

[16] Chance, Love and Logic, p. 195.

[17] *Ibid.*, p. 177.

[18] *Ibid.*, p. 162.

It is not necessary to follow Peirce in all his ideas, but it is advantageous to recognize with him and a long line of predecessors that a purely mechanistis philosophy does not cover all the phenomena of nature. It is not necessary to slacken our endeavors to explain, as duly caused, all phenomena which may be so explained, and we are entirely sympathetic with the mechanistic developments in genetics, physiology, and biology in general; but it is important to realize that there will always remain points at which such explanations break down, and recourse must be had to a classificatory procedure such as has been urged relative to meteorology. It is possible that the present difficulties in reconciling the new quantum phenomena with the old classical physics are in part due to the fact that physicists have long since abandoned the classificatory method and are too loath to return to it.